IV. Conclusion

The FDFD method, which had been shown to yield reliable results for nearly arbitrarily shaped structures, was expanded in order to allow the analysis of structures with lossy materials. A simple coplanar short circuit was investigated with this method. In order to verify the improved FDFD method the transmission line parameters (β, α, and Z_W) of a homogeneous waveguide were compared with results obtained by a highly sophisticated and well-proven model. Good agreement was observed. The effects already known from homogeneous coplanar waveguides were also observed for the short circuit, e.g., the external inductance changes significantly for low frequencies. This is due to conductor losses whereas the influence of the dielectric losses is comparatively small. Finally, a simple model was presented that describes the parasitics of a lossy short end with good accuracy.

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A New Extraction Method for Noise Sources and Correlation Coefficient in MESFET

Jong-Hee Han and Kwyro Lee

***Abstract*—A new extraction method for noise sources and correlation coefficient in the noise equivalent circuit of GaAs metal semiconductor field effect transistor (MESFET) is proposed. It is based on the linear regression, which allows us to extract physically meaningful parameters from the measurement in a systematic and straightforward way. The confidence level of the measured data can also be easily examined from the linearity, *y*-intercept of the linear regression, and the scattering from the regression line. Furthermore, it is found that the time constant of correlation coefficient whose value is almost the same as that of the transconductance should be considered to model noise parameters accurately. The calculated values of minimum noise figure, optimum impedance, and noise resistance using above approach, show excellent agreement with measurement for a typical MESFET device studied in this paper.**

I. Introduction

The noise characteristics of a linear two-port system can be completely described by 2-port parameters such as Y, Z, S-parameters and the additional four noise parameters. There are several equivalent sets of these four noise parameters, depending on how to represent the two-port circuit [1], [2]. In addition to the high frequency equivalent circuit parameters, the most important noise parameters from the viewpoint of circuit designers are the minimum noise figure, the optimum impedance (real and imaginary parts), and the noise resistance. They are essential in low noise circuit design. However, it is difficult to incorporate them into the small signal equivalent circuit and the physical relationship between them is not obvious. Therefore, purely empirical representations such as Y- or H-representations are popularly used in the noise equivalent circuit.

Recently, it is reported by Pospieszalski [3] that in the case of H-representation of metal semiconductor field effect transistor (MESFET), the correlation between the noise voltage source at the gate and the noise current source at the drain can safely be ignored in expressing the measured noise characteristics of MESFET with a decent accuracy. Granted it is true in the intrinsic device, the parasitic gate-source capacitance, C_{gsp}, invokes the correlation between gate noise voltage and drain noise current sources [4]. Since it is very difficult to discriminate C_{gsp} from the intrinsic capacitance C_{gsi} by S-parameter measurement, and according to Anholt's estimation of C_{gsp} [5], C_{gsp} is comparable to C_{gsi} especially in the case of low current where the better noise characteristics is obtained, the correlation coefficient has to be considered in the noise equivalent circuit.

We found that the increase of the imaginary part of correlation coefficient with frequency affects the noise characteristics considerably at high frequencies as will be shown in Section IV. The time constant of correlation coefficient is introduced to explain this. But this introduction does not add the number of parameters because the time delay of transconductance is found to be almost the same as time constant of the correlation coefficient. The physical reason for this justification will be explained in Section II.

Manuscript received June 29, 1995; revised November 27, 1995. This work was supported in part by the Agency for Defense Development.

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Publisher Item Identifier S 0018-9480(96)01555-4.

Usually a numerical optimization technique is used for extracting the noise sources from the measured noise characteristics by minimizing the error between the measurement and model. Though conceptually accurate, numerical optimization often gives the wrong answer due to the local extrema. Furthermore, it is hard to figure out the confidence level of the extraction result. In this paper, we propose a new method to determine the noise sources from the measurement by linear regression. This allows us not only to extract parameters in a systematic and straightforward way, but also to figure the confidence level of the extraction easily.

In Section II, the theoretical background for extracting the noise parameters in noise equivalent circuit is described. The extraction examples from a typical MESFET device are shown in Section III, and compared with the measurement in Section IV. A conclusion follows in Section V.

II. Noise Modeling of MESFET

Fig. 1 shows MESFET small signal equivalent circuit and the H-representation noise equivalent circuit with the correlation coefficient, C_H. Here, C_H is defined as

$$C_H = \frac{\langle v_{gn} i_{dn}^* \rangle}{\sqrt{\langle v_{gn}^2 \rangle \langle i_{dn}^2 \rangle}} \tag{1}$$

which is a complex number in general. Note that C_{gs} is the sum of C_{gsp} and C_{gsi}. In order to calculate the noise characteristics in the form of minimum noise figure, optimum impedance, and noise resistance, the input equivalent noise voltage, v_{in}, and the noise current sources, i_{in}, are derived in the intrinsic equivalent circuit (inside the dotted box in Fig. 1). v_{in} is the input equivalent noise voltage source which generates the same output noise when the gate is short to ground in AC, given by

$$v_{in} = v_{gn} + \frac{1 + j\omega C_{gs} R_i}{g_m e^{-j\omega\tau}} i_{dn}. \tag{2}$$

i_{in} is the input equivalent noise current generating the same output noise when the gate is open in AC, which can be written as

$$i_{in} = -\frac{j\omega C_{gs}}{g_m e^{-j\omega\tau}} i_{dn}. \tag{3}$$

Noise conductance, G_n can be obtained by normalizing $\langle i_{in}^2 \rangle$ by $4kT\Delta f$ as

$$G_n = \frac{1}{4kT\Delta f} \frac{\omega^2 C_{gs}^2}{g_m^2} \langle i_{dn}^2 \rangle. \tag{4}$$

Here, k is the Boltzmann constant, T the absolute temperature, and Δf the bandwidth. Last, correlation impedance between v_{in} and i_{in}, Z_c, defined by $\langle v_{in} i_{in}^* \rangle / \langle i_{in}^2 \rangle$ can be derived using (2)–(3) as follows

$$Z_c = -R_i + \frac{j}{\omega C_{gs}} + \frac{j g_m e^{-j\omega\tau} C_H}{\omega C_{gs}} \sqrt{\frac{\langle v_{gn}^2 \rangle}{\langle i_{dn}^2 \rangle}}. \tag{5}$$

In general, C_H is a complex number, but usually C_H can be assumed to have only the real part in MESFET as we discussed before. This assumption is plausible at low frequency, but the imaginary part of C_H increases at higher frequency. Ignoring the imaginary part of C_H gives rise to the large discrepancy in noise characteristics between measurement and model especially at high frequency as will be shown later. Fig. 2 shows the measured magnitude and angle of C_H. Here, C_H is obtained after de-embedding R_g, R_s, R_d, C_{gd} and the parasitic inductances using Hillbrand's method [6]. Fig. 2 shows that the magnitude of C_H is almost constant independently of the frequency. However, the angle of C_H is almost proportional to the frequency. This linear dependency of the angle can be expressed by the time constant, i.e., $e^{j\omega\tau_H}$. It is found that the measured τ_H is almost the same as the time delay of transconductance, g_m. For instance, Fig. 2 shows that τ_H is 6 psec, while g_m delay is 4 psec at the same bias condition. The origin of g_m delay is said to be due to the transit time effect in the saturation region of the channel [7]. In MESFET's, it is almost agreed that the v_{gn} and i_{dn} have the same origin, i.e., due to the noise generated at some point in the channel. Then it is natural to speculate that the i_{dn} has some delay with respect to v_{gn}, whose amount is comparable to g_m delay. Therefore, it is a good first-order approximation to take g_m delay, τ, measured from S-parameters as the time constant of C_H. Then we can write C_H as $C_H e^{j\omega\tau}$. Using this, the correlation impedance (5) can now be rewritten as

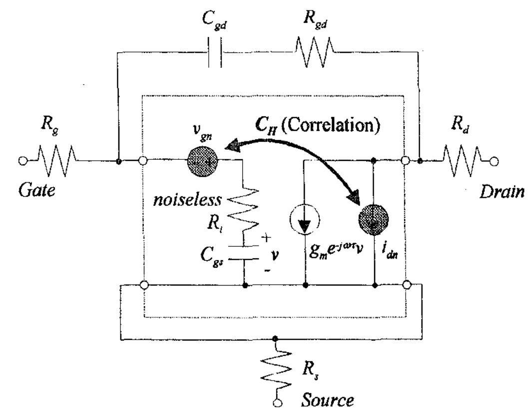

Fig. 1. A noise equivalent circuit in H-representation for MESFET. The parasitic elements outside the dotted box on the noise characteristics are de-embedded before the extraction of noise sources and correlation coefficient. Note that C_{gs} is the sum of the intrinsic and parasitic capacitances.

$$Z_c = -R_i + \frac{j}{\omega C_{gs}} \left(1 + g_m C_H \sqrt{\frac{\langle v_{gn}^2 \rangle}{\langle i_{dn}^2 \rangle}} \right). \tag{6}$$

The imaginary part of the optimum impedance, X_{opt}, is the same as that of Z_c, given by

$$X_{opt} = \frac{1}{\omega C_{gs}} \left(1 + g_m C_H \sqrt{\frac{\langle v_{gn}^2 \rangle}{\langle i_{dn}^2 \rangle}} \right). \tag{7}$$

Note that X_{opt} is inversely proportional to ωC_{gs}, and that its proportional constant is dependent on C_H. On the other hand, the real part of the optimum impedance, R_{opt}, is

$$R_{opt}^2 = R_i^2 + \frac{R_n - |Z_c^2| G_n}{G_n}. \tag{8}$$

Here R_n is the $\langle \mathbf{v}_n^2 \rangle$ normalized by $4kT\Delta f$, which is derived from (2) as

$$R_n \approx R_{gn} + \frac{G_{dn}}{g_m^2} + 2\frac{C_H}{g_m} \sqrt{R_{gn} G_{dn}}. \tag{9}$$

In (9) R_{gn} and G_{dn} are $\langle \mathbf{v}_{gn}^2 \rangle$ and $\langle \mathbf{i}_{dn}^2 \rangle$ normalized by $4kT\Delta f$, respectively. In deriving (9), the second order term of frequency is neglected. Rewriting R_{opt}^2 in (8) using (9) leads

$$R_{opt}^2 = R_i^2 + (1 - C_H^2) \frac{\langle v_{gn}^2 \rangle}{\langle i_{dn}^2 \rangle} \left(\frac{\omega_T}{\omega} \right)^2 \tag{10}$$

where ω_T is the unit current gain frequency given by g_m / C_{gs}. It is very interesting to notice that $R_{opt}^2 - R_i^2$ is inversely proportional to the square of frequency. Finally, the minimum noise figure can be written as

$$NF_{\min} = 10 \log \left(1 + 2G_n (R_{opt} - R_c) \right) \tag{11}$$

where R_c is the real part of Z_c, and equal to R_i in this model.

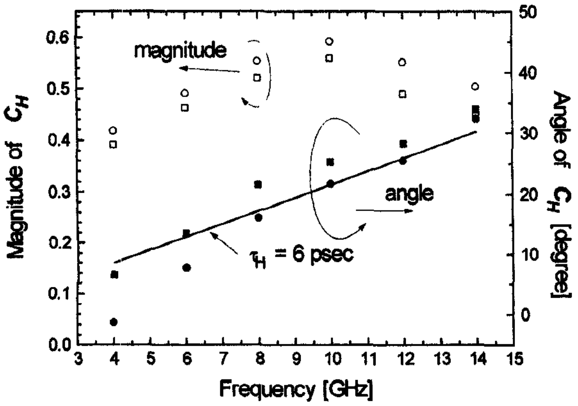

Fig. 2. The frequency dependence of the magnitude and the angle of measured correlation coefficient in H-representation. Squares and circles are experimental measurement result at $V_{DS} = 2.0$ V and 3.0 V, respectively. V_{GS} is -0.8 V.

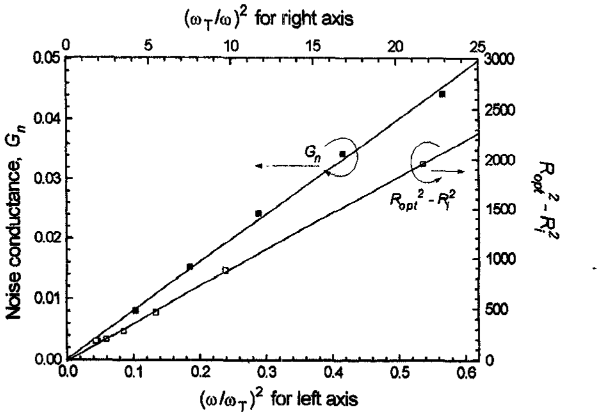

Fig. 3. G_n versus $(\omega/\omega_T)^2$ and $(R_{opt}^2 - R_i^2)$ versus $(\omega_T/\omega)^2$ plot at $V_{GS} = 0.0$ V and $V_{DS} = 3.0$ V. Note the y-intercept is zero in agreement with (4) and (12).

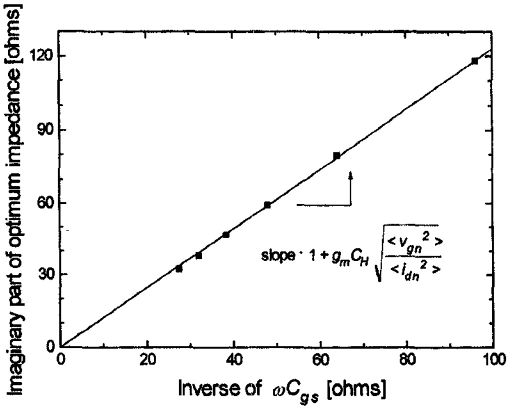

Fig. 4. X_{opt} versus $\frac{1}{\omega C_{gs}}$ plot at $V_{GS} = 0.0$ V and $V_{DS} = 3.0$ V. Note that the y-intercept is zero in agreement with (9).

III. Extraction Example of Noise Sources from Measurement

In this section, we show a typical extraction example. The measured device is an ion-implanted GaAs MESFET, whose geometry is 0.5×300 μm^2. First, the small signal equivalent circuit parameters in Fig. 1 are extracted from "hot-cold" S-parameter measurement [8]. Remaining three noise parameters, $\langle v_{gn}^2 \rangle$, $\langle i_{dn}^2 \rangle$, and C_H are extracted from the frequency dependent noise measurement. As derived in Section II, the combinations of these three parameters have linear relations with G_n (4), X_{opt} (7), and R_{opt}^2 (10), respectively. Therefore, we can extract them from the linear regression of three plots, i.e., G_n versus $(\omega/\omega_T)^2$, X_{opt} versus $1/\omega C_{gs}$, and $(R_{opt}^2 - R_i^2)$ versus $(\omega_T/\omega)^2$ after de-embedding the parasitic elements. The inherent advantages of the linear regression are: 1) there is no danger to go into local extrema; and 2) the confidence level of the extraction results can be checked easily from the linearity of the plots, i.e., slope, y-intercept, and the scattering from the regression line.

The examples of the extraction plots are shown in Figs. 3 and 4. The measurement shows excellent linearity and the scattering of measured data from the regression line also is very small. Note that the slope of X_{opt} versus $1/\omega C_{gs}$ is not one. In the Pospieszalski's model, i.e., when C_H is zero, X_{opt} should be exactly the same as $1/\omega C_{gs}$. But, as shown in Fig. 4, the slope is 1.2. This means the correlation between v_{gn} and i_{dn} should not be ignored.

IV. Comparison of Model with Measurement

The minimum noise figure, the optimum impedance, and the noise resistance of our proposed model are compared with those of measurement. In addition to our model, we compare two more cases. One is the case when C_H is assumed to have only real part in the noise parameter calculation, and the other is when C_H is equal to 0, i.e., Pospieszalski's model. The noise sources in Pospieszalski's model are extracted from the slopes in the plot of G_n versus $(\omega/\omega_T)^2$ and $(R_{opt}^2 - R_i^2)$ versus $(\omega_T/\omega)^2$. That is, the first slope gives $\langle i_{dn}^2 \rangle$ (4) and the second gives $\langle v_{gn}^2 \rangle$ over $\langle i_{dn}^2 \rangle$ (10). Comparison is performed in Fig. 5 at $I_{DS} = 15$ mA ($V_{GS} = -0.8$ V) and $V_{DS} = 3.0$ V. Fig. 5 compares the noise characteristics of the extrinsic device including parasitic elements using above mentioned three different models.

In the case of real C_H, we can see the calculated minimum noise figure is far off the measured one. If the angle of C_H is zero, i.e. in the case of real C_H, the real part of correlation impedance increases. This makes the minimum noise figure decrease considerably at high frequencies (see (11)). In the case of Pospieszalski's model, the calculated minimum noise figure and R_{opt} agree reasonably well with the measurement. But, the calculated X_{opt} and R_n show appreciable discrepancy with measurement. This is because X_{opt} when $C_H = 0$ is the inverse of ωC_{gs}, while it increases when C_H is not zero (see (7)). And R_n when $C_H = 0$ is smaller than that when C_H is not zero due to the absence of the third term in (9). The Fig. 5 show that our model has much better accuracy than the other two models in explaining frequency dependence of all four noise parameters.

V. Conclusion

A new extraction method for noise sources and correlation coefficient in the noise equivalent circuit of GaAs MESFET has been proposed. Since the extraction of noise sources and correlation coefficient is based on the linear regression, it allows us not only to extract physically meaningful parameters from the measurement in a systematic and straightforward way, but also to examine easily the confidence level of the measured data from the linearity of three plots and the scattering from the regression lines. And, it is found that the time constant of correlation coefficient whose value can be approximated as transconductance delay should be considered. The comparison of noise parameters between our model and measurement shows good agreements at high and low drain current bias for a typical MESFET device studied in this paper.

Acknowledgment

The authors would like to thank Dr. C. T. Kim and Dr. W. S. Cho at LG Electronics, Inc. for their help in noise measurement.

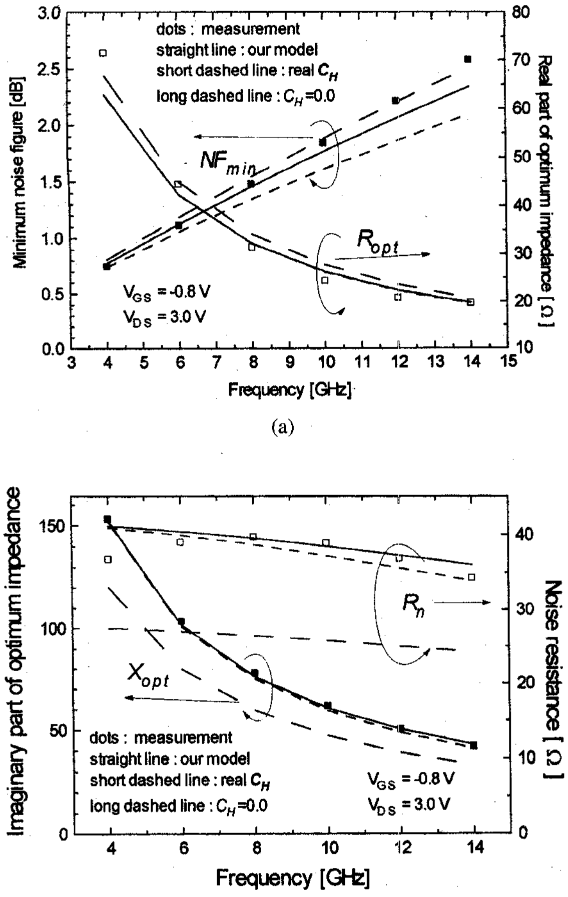

Fig. 5. (a) The minimum noise figure and the real part, and (b) the imaginary part of the optimum impedance and the noise resistance at $V_{GS} = -0.8$ V and $V_{DS} = 3.0$ V.

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The Noise-Tee—A Lightwave Device for Microwave Noise Measurements

Rob F. M. van den Brink

Abstract—**An innovative lightwave method is proposed to insert noise in electronic circuits in favor of microwave noise measurements. The proposed noise-tee has attractive additional features compared to the use of 50 Ω noise sources: 1) The inserted noise level and noise bandwidth is continuously variable over a wide dynamic range; 2) The wideband scaling accuracy of this level, relative to a pre-calibrated level, equals the accuracy of simple dc-current measurements; 3) Level-induced impedance variations are negligible, compared to the 20% impedance variation of a commonly used microwave noise source; and 4) Noise-tees enable the realization of 100% reflective noise sources, in favor of two-port noise-parameter measurements.**

I. INTRODUCTION

The characterization of equivalent input noise of amplifiers and systems is of basic importance for various applications. When white noise is supplied to the input of an amplifier under test, the equivalent input noise of that amplifier can easily be determined by comparing its unknown level with the known level of the supplied white noise. This ratio-approach is well known and recommended by IRE/IEEE standards [1]–[6], [8]. Various microwave noise sources are commercially available for this purpose, but are commonly restricted to two fixed noise levels ("hot" and "cold") and to a "fixed" output impedance (usually 50) that varies with the switched noise level. Undesired variations of 20% have been observed in practice, as demonstrated in Fig. 4. They deteriorate the accuracy of simple equivalent noise measurements at a specified source impedance. Mathematical correction for this effect will improve the measurement accuracy [3], [6], [7] but requires additional measurements to determine the associated gain and noise variation of the amplifier under test.

We propose the use of p-i-n photodiodes to insert white noise in a measurement setup, generated by a *synthetic noise generator*. This is a new lightwave instrument, as reported in [9], [11], and [12], having a fiber-optic output to illuminate the photodiode. Insertion of levels 40 dB above the thermal noise level of 50-Ω resistors is feasible.

The proposed method has attractive additional features, compared to conventional 50-Ω noise sources, including:

1) Continuous variation of inserted noise level, over a wide dynamic range using lightwave attenuation.
2) Simple relative scaling of inserted noise level, using dc-current measurements.
3) Variable bandwidth of inserted noise, ranging from several MHz to the photodiode bandwidth.
4) Negligible disturbance of source impedance, when insertion level varies over a wide dynamic range.

This paper discusses the application of p-i-n diodes in a "noise-tee" configuration for inserting noise in a noise measurement setup.

II. CIRCUIT DESCRIPTION OF A NOISE-TEE

A noise-tee is a two-port configuration for inserting lightwave generated noise, similar to the insertion of dc-bias-currents using

Manuscript received July 26, 1995; revised November 27, 1995.

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Publisher Item Identifier S 0018-9480(96)01556-6.